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5. Putting $a_1 = a_2 = a$, $b_1 = b_2 = b$, $c_1 = c_2 = c$, $d_1 = d_2 = d$, in (2) we have

$$\begin{vmatrix} a^2-b^2-c^2-d^2 & 2ab & 2ac & 2ad \\ 2ab & b^2-c^2-d^2-a^2 & 2bc & 2bd \\ 2ac & 2bc & c^2-d^2-a^2-b^2 & 2cd \\ 2ad & 2bd & 2cd & d^2-a^2-b^2-c^2 \end{vmatrix} = -(a^2+b^2+c^2+d^2)^4$$

and from this, by making $d = 0$, there results

$$\begin{vmatrix} a^2-b^2-c^2 & 2ab & 2ac \\ 2ab & b^2-c^2-a^2 & 2bc \\ 2ac & 2bc & c^2-a^2-b^2 \end{vmatrix} = (a^2+b^2+c^2)^3$$

and thence in the same way

$$\begin{vmatrix} a^2+b^2 & 2ab \\ 2ab & b^2-a^2 \end{vmatrix} = -(a^2+b^2)^2,$$

the identity well known in connection with Euc. I. 47, giving the sum of two squares as a square.

THE BITANGENTIAL.

BY WILLIAM E. HEAL, MARION, INDIANA.

THE curve which passes through the points of contact of bitangents of a given curve is called the *bitangential* of that curve.

Such a curve may be determined by the method of problem 331, ANALYST. It is, however, desirable to obtain a curve of lower order, and for this purpose Salmon has given two methods in Higher Plane Curves.

Let us put

$$A = \begin{vmatrix} \frac{d^2u}{dy^2} & \frac{d^2u}{dz\,dy} \\ \frac{d^2u}{dy\,dz} & \frac{d^2u}{dz^2} \end{vmatrix}, \quad B = \begin{vmatrix} \frac{d^2u}{dx^2} & \frac{d^2u}{dz\,dx} \\ \frac{d^2u}{dx\,dz} & \frac{d^2u}{dz^2} \end{vmatrix}, \quad C = \begin{vmatrix} \frac{d^2u}{dx^2} & \frac{d^2u}{dydx} \\ \frac{d^2u}{dxdy} & \frac{d^2u}{dy^2} \end{vmatrix},$$

$$D = \begin{vmatrix} \frac{d^2u}{dx\,dz} & \frac{d^2u}{dz\,dy} \\ \frac{d^2u}{dx^2} & \frac{d^2u}{dxdy} \end{vmatrix}, \quad E = \begin{vmatrix} \frac{d^2u}{dxdy} & \frac{d^2u}{dx\,dz} \\ \frac{d^2u}{dy^2} & \frac{d^2u}{dy\,dz} \end{vmatrix}, \quad F = \begin{vmatrix} \frac{d^2u}{dy\,dz} & \frac{d^2u}{dxdy} \\ \frac{d^2u}{dz^2} & \frac{d^2u}{dx\,dz} \end{vmatrix};$$

$$H = \begin{vmatrix} \frac{d^2u}{dx^2} & \frac{d^2u}{dxdy} & \frac{d^2x}{dx\,dz} \\ \frac{d^2u}{dydx} & \frac{d^2u}{dy^2} & \frac{d^2u}{dy\,dz} \\ \frac{d^2u}{dz\,dx} & \frac{d^2u}{dz\,dy} & \frac{d^2u}{dz^2} \end{vmatrix};$$

$$\Theta = A\frac{dH^2}{dx^2} + B\frac{dH^2}{dy^2} + C\frac{dH^2}{dz^2} + 2D\frac{dH}{dy}\frac{dH}{dz} + 2E\frac{dH}{dz}\frac{dH}{dx} + 2F\frac{dH}{dx}\frac{dH}{dy},$$

$$\Phi = A\frac{d^2H}{dx^2} + B\frac{d^2H}{dy^2} + C\frac{d^2H}{dz^2} + 2D\frac{d^2H}{dy\,dz} + 2E\frac{d^2H}{dz\,dx} + 2F\frac{d^2H}{dxdy},$$

$$\Psi = A\frac{d^2H}{dx^2} + B\frac{d^2\Phi}{dy^2} + C\frac{d^2\Phi}{dz^2} + 2D\frac{d\Phi}{dy}\frac{d\Phi}{dz} + 2E\frac{d\Phi}{dz}\frac{d\Phi}{dx} + 2F\frac{d\Phi}{dx}\frac{d\Phi}{dy},$$

$$\chi = A\frac{dH}{dx}\frac{d\Phi}{dx} + B\frac{dH}{dy}\frac{d\Phi}{dy} + C\frac{dH}{dz}\frac{d\Phi}{dz} + D\left(\frac{dH}{dy}\frac{d\Phi}{dz} + \frac{dH}{dz}\frac{d\Phi}{dy}\right)$$
$$+ E\left(\frac{dH}{dz}\frac{d\Phi}{dx} + \frac{dH}{dx}\frac{d\Phi}{dz}\right) + F\left(\frac{dH}{dx}\frac{d\Phi}{dy} + \frac{dH}{dy}\frac{d\Phi}{dx}\right).$$

Then the equation of the bitangential of a quartic is

$$\Theta = 3H\Phi.$$

(Salmon's Higher Plane Curves, page 337.)

In attempting to find the bitangential of a quintic Salmon says (Higher Plane Curves, page 339), "In order to form the bitangential curve of a quintic, the quantity to be calculated is $(27q_2q_5-5q_3q_4)^2 = 5(4q_3^2-9q_2q_4)(5q_4^2-12q_3q_5)$, a quantity containing $\alpha\beta\gamma$ in the sixth order, and which it is necessary, by the help of the equation of the curve, to show is divisible by R^6.

"Now in virtue of a formula already obtained we have

$$4q_3^2 - 9q_2q_4 = R^2(4\Theta - 9H\Phi).$$

It is also easy to show that $27q_2q_5 - 5q_3q_4$ and $5q_4^2 - 12q_3q_5$ are each divisible by R_5; but I have not been able to carry the reduction further."

In attemting to complete the solution of this problem I arrived at the following result:

The equation of the bitangential of a quintic is

$$529H^2\Psi - 270H\Phi\chi + 25\Phi^2\Theta = 5(4\Theta - 9H\Phi)(5\Phi^2 - 12\chi),$$

a curve of the forty-eighth order, as it should be.

I have not been able to *prove* this result, and submit it to the readers of the ANALYST hoping that some one may be more successful than myself.

It will be observed that we have used trilinear coordinates and that the equation of the curve is

$$u = f(x, y, z) = 0,$$